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ESTIMATING THE TOTAL EFFECT ON A FORMATION DURING BOREHOLE DRILLING

A. A. Kashevarov

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A stationary mathematical model describing the time-integrated effect on an oil-saturated reservoir during drilling is considered. Calculated results are compared with the solution of the problem in an exact nonstationary formulation. The formation of an invaded zone in straight borehole drilling in water- and oil-saturated reservoirs is studied by numerical modeling.

Key words: *modeling, invaded zone, reservoir, oil-saturation, salt concentration, borehole.*

Introduction. During borehole drilling, the high pressure difference causes the drilling mud to invade the oil-saturated reservoir and displace the interstitial fluids — water and oil. As the mud filter cake builds up on the borehole wall, the filtration process is rapidly decelerated, and, after cessation of drilling, the borehole and formation pressures are equalized. In the drilling mud and formation water, the salt concentrations, as a rule, are different. As a result, an invaded zone with changed electric-conductivity properties is formed near the borehole. Borehole logging measurements shows changes in the physical properties in the invaded zone, which allow oil-saturated reservoirs to be identified.

A hydrodynamic model for the formation of the invaded zone allows one to estimate its characteristics and find the dependence between the hydrophysical parameters of reservoirs and electromagnetic sounding results. However, numerical modeling of the borehole drilling process and mud filtrate invasion of a reservoir requires great computational efforts. In some cases, information on the invasion dynamics is not required and it is sufficient to know only the invaded-zone characteristic at the end of the drilling process. It is proposed to solve such problems using a stationary model for the time-integrated hydrodynamic head, which provides an estimate of the total effect of the excess borehole pressure on the reservoir during drilling. This stationary model predicts the drilling mud loss at various hole drilling intervals, the mud cake thickness, and the depth of the invaded zone.

Nonstationary Hydrodynamic Model for the Invaded Zone. In borehole drilling, the main physical factors determining the processes in the near-borehole region are drilling mud circulation, buildup and destruction of the mud cake in various regions of reservoirs, filtration of fluids (water and oil), and salt transport processes.

We consider a nonstationary hydrodynamic model for the formation of an invaded zone [1]. In cylindrical coordinates, two-phase filtration is described by the Buckley–Leverett equations with the additional condition imposed on the sum of the water and oil saturations $S_{\rm w} + S_{\rm oil} = 1$. The transport equations for the moving phases are written as follows [2, 3]:

$$\frac{\partial}{\partial t}\,(rmS_\xi) = \frac{\partial}{\partial r}\Big(rk_\xi\,\frac{\partial h}{\partial r}\Big) + \frac{\partial}{\partial z}\Big(rk_\xi\,\frac{\partial h}{\partial z}\Big), \qquad (r, z) \in \Omega = \{r_b < r < L,\ 0 < z < L_z\}. \tag{1}$$

Here the subscript ξ refers to the fluid (ξ = w for water and ξ = oil for oil), the z coordinate axis is directed downward and coincides with the axis of a borehole of radius r_b, $h = p/(\rho g) - z$ is the hydrodynamic head, p is the pressure, ρ is the density of the interstitial fluid, $m = m_0 + \varepsilon(p - p_0)$ is the reservoir porosity, ε is the compressibility; $k_{\rm w} = k_0 S_{\rm w}^{n_1}$ and $k_{\rm oil} = \mu_0 k_0 S_{\rm oil}^{n_2}$ are functions that define the phase permeability of water and

Lavrent'ev Institute of Hydrodynamics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090; kash@hydro.nsc.ru. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 49, No. 2, pp. 113–122, March–April, 2008. Original article submitted March 6, 2007.

oil, respectively, $\mu_0 = \mu_{\rm w}/\mu_{\rm oil}$ is the ratio of the viscosities of water and oil, and $k_0 = k/\mu_{\rm w}$ (k is the formation permeability).

The boundary and initial conditions are specified under the assumption that, at the initial time, the pressure head over the entire reservoir is equal to the formation pressure head $h_f = \text{const}$:

$$h\Big|_{r=L} = h_f, \qquad \frac{\partial h}{\partial z}\Big|_{z=0,L_z} = 0, \qquad h\Big|_{t=0} = h_f.$$

On the left boundary ($r = r_b$), which corresponds to the borehole wall, the condition of the third kind is specified, which models the interaction of the borehole with the formation [1]. The borehole bottom [$z = l_b(t)$] moves into the depth of the formation at a rate equal to the drilling rate v_b:

$$-q = (k_{\rm w} + k_{\rm oil})\,\frac{\partial h}{\partial r}\Big|_{r=r_b} = \beta(h\Big|_{r=r_b} - h_b), \quad 0 \leqslant z \leqslant l_b, \qquad \frac{\partial h}{\partial r}\Big|_{r=r_b} = 0, \quad l_b < z \leqslant L_z. \tag{2}$$

If, in a certain region of the boundary, the flow is directed into the region Ω, one needs to specify the water-saturation value in this region:

$$S_{\rm w}\Big|_{r=r_b} = 1, \qquad S_{\rm w}\Big|_{r=L} = S_f, \qquad S_{\rm w}\Big|_{t=0} = S_f. \tag{3}$$

For the borehole, the excess of the hydrodynamic pressure head $h_b(t)$ over the initial pressure head in the reservoir is specified. In the following, we set $h_f = 0$.

Salt transport is modeled using the transport equation for a conservative impurity [2]. The relative concentration of salts C transported by water obeys the equation

$$\begin{gathered}\frac{\partial}{\partial t}\,(rmS_{\rm w}C) + \frac{\partial}{\partial r}\,(v_rC) + \frac{\partial}{\partial z}\,(v_zC) = 0,\\ v_r = -rk_{\rm w}\,\frac{\partial h}{\partial r}, \qquad v_z = -rk_{\rm w}\,\frac{\partial h}{\partial z}, \qquad C\Big|_{t=0} = C_f.\end{gathered} \tag{4}$$

Boundary conditions for the salt transport equation are specified only for the region in which water passes into the formation through the boundary of the modeling domain. In these regions of the boundary, the concentration is set equal to the formation water concentration ($C|_{r=L} = C_f$) or the mud filtrate concentration ($C|_{r=r_b} = C_b$).

Let us consider a submodel for the increase in the mud cake thickness which is suitable for water-based clay drilling muds. The cake thickness $d(z,t)$ enters the parameter $\beta = (\beta_0^{-1} + d/k_c^0)^{-1}$, which defines water exchange between the borehole and the formation and takes into account the flow coefficient of the clogging layer formed at the initial time of drilling (β_0^{-1}) and the additional resistance due to the presence of the mud cake (d/k_c^0) on the borehole wall ($k_c^0 = k_c/\mu_{\rm w}$, where k_c is the permeability of the mud cake). The rate of increase in the mud cake thickness is proportional to the rate of filtration ($q = -k_0 h_r|_{r=r_b}$) of the drilling mud from the borehole into the formation and is given by the equation [1, 3]

$$d'_t = -\alpha q. \tag{5}$$

Here the parameter $\alpha = \delta/[(1-\delta)(1-m_c)]$ depends on the porosity of the mud cake m_c and the fraction of clay particles δ in the drilling mud.

For numerical implementation, instead of the equation for the oil saturation we used the equation for the total circulation flow [1] obtained by summation of the equations for water and oil saturations (1):

$$\frac{\partial}{\partial t}\,(rm) = \frac{\partial}{\partial r}\,\Big(r(k_{\rm w} + k_{\rm oil})\,\frac{\partial h}{\partial r}\Big) + \frac{\partial}{\partial z}\,\Big(r(k_{\rm w} + k_{\rm oil})\,\frac{\partial h}{\partial z}\Big). \tag{6}$$

The beginning of the calculation corresponds to the time the drilling tool penetrates a formation bounded at the top and bottom by impermeable clay layers. It should be noted that, after the drilling tool is removed from the reservoir, the effect on the formation continues because, during borehole drilling and washing due to drilling mud circulation, the borehole pressure is much higher than the formation pressure.

The numerical calculations were performed using implicit finite-difference schemes and the iterative variable direction method [4]. For the transport equations, the countercurrent approximation [5] was used.

Stationary Hydrodynamic Model for Estimating the Total Borehole Effect on Formations during Drilling. We make a number of assumptions to simplify the initial nonstationary model of the invaded zone. Under standard borehole drilling conditions, a relatively small volume of filtrate invades the formation because of a rapid increase in the mud cake thickness. Therefore, the depth of the invaded zone, in which there is a change in the formation-fluid distribution, is also small and, as a rule, does not exceed 0.4–0.6 m. However, the change in the formation pressure head due to the excess borehole pressure occurs at a significant distance from the borehole and depends on the formation compressibility and formation rock permeability. The radius of influence of the borehole is several tens (for $k = 1$–10 millidarcy) to several hundreds (for $k > 100$ millidarcy) meters (1 millidarcy $\approx 1.02 \cdot 10^{-15}$ m^2), depending on the formation permeability. Consequently, the change in the formation permeability in the invaded zone has an insignificant influence on the pressure head distribution in the reservoir; hence, instead of Eq. (6), we can use the linear equation

$$\frac{\partial}{\partial t}(rm) = \frac{\partial}{\partial r}\Big(rk_f\,\frac{\partial h}{\partial r}\Big) + \frac{\partial}{\partial z}\Big(rk_f\,\frac{\partial h}{\partial z}\Big), \qquad k_f = k_{\mathrm{w}}(S_f) + k_{\mathrm{oil}}(1 - S_f). \tag{6a}$$

For strata, constant permeabilities are specified according to the initial values of the water saturation S_f. It should be noted that this model provides the most adequate description of the pressure head variation for moderately permeable and highly permeable formations. For drilling of water-saturated reservoirs ($S_f = 1$) using water-based drilling muds, Eq. (6a) is satisfied exactly.

We consider the case where the drilling tool has penetrated below the interval being modeled, i.e., the borehole bottom is below the formation bottom level. This is reasonable in the case of high drilling rates (8–10 m/h), where the time of drilling of thin formations can be ignored. Below, we consider the possibility of taking into account the influence of the borehole on the formation during drilling.

In view of the assumptions made above, Eq. (6a) can be integrated with respect to time over the entire period of drilling $[0,T]$. Then, for the time-integrated pressure head $H = \int\limits_0^T h\,dt$, we obtain the equation

$$\frac{\partial}{\partial r}\Big(rk_f\,\frac{\partial H}{\partial r}\Big) + \frac{\partial}{\partial z}\Big(rk_f\,\frac{\partial H}{\partial z}\Big) = \Phi = \varepsilon r(p(r,z,T) - p_0). \tag{7}$$

After some hours of drilling, a mud cake is formed on the formation wall, resulting in a sharp drop in the formation pressure. After the cessation of drilling ($t \geqslant T$) and drilling fluid circulation, a hydrostatic pressure distribution is established in the borehole. According to the reservoir drilling conditions, the hydrostatic pressure in the borehole and formation are close. Therefore, the right side of Eq. (7) can be set equal to zero with high accuracy: $\Phi = 0$. We obtain a stationary equation which approximately describes the integrated-pressure distribution over the entire modeling domain.

The boundary nonpenetration conditions on the formation top and bottom and the condition of the first kind on the right boundary ($r = L$) are written

$$\frac{\partial H}{\partial z}\Big|_{z=0,L_z} = 0, \qquad H\Big|_{r=L} = 0. \tag{8}$$

The boundary condition on the borehole wall is written in terms of the new required function H. In view of relation (5), the first condition in (2) is represented as

$$d'_t = \alpha\beta(h_b - h|_{r=r_b}) = \alpha(d/k_c^0 + \beta_0^{-1})^{-1}(h_b - h|_{r=r_b}).$$

Integration of this ordinary differential equation yields

$$\Big(\frac{d}{k_c^0} + \beta_0^{-1}\Big)^2 = \frac{2\alpha}{k_c^0}\,(H_b - H|_{r=r_b}) + \beta_0^{-2}, \qquad H = \int\limits_0^T h\,dt, \qquad H_b = \int\limits_0^T h_b\,dt.$$

This implies the equation

$$d = k_c^0\Big(\sqrt{(2\alpha/k_c^0)(H_b - H\big|_{r=r_b}) + \beta_0^{-2}} - \beta_0^{-1}\Big).$$

In addition, from (5) we have

$$Q = \int\limits_0^T q\,dt = -k_f H_r\Big|_{r=r_b} = \frac{d}{\alpha}.$$

From this, using the equation for d, we write the following boundary condition on the borehole wall:

$$-k_f H_r\Big|_{r=r_b} = (k_c^0/\alpha)\Big(\sqrt{(2\alpha/k_c^0)(H_b - H\Big|_{r=r_b}) + \beta_0^{-2}} - \beta_0^{-1}\Big). \tag{9}$$

Thus, in the case $\Phi = 0$, for the integrated pressure head H, we obtain the closed boundary-value problem (7)–(9).

In numerical implementation of the stationary model (7)–(9), we used an iterative algorithm that reduced the nonlinear problem to a sequence of linear problem. For the nth iterative step, the following boundary-value problem was solved:

$$\frac{\partial}{\partial r}\Big(rk_f\,\frac{\partial H^n}{\partial r}\Big) + \frac{\partial}{\partial z}\Big(rk_f\,\frac{\partial H^n}{\partial z}\Big) = \frac{H^n - H^{n-1}}{\tau}, \quad (r,z) \in \Omega = \{r_b < r < L,\ 0 < z < L_z\},$$

$$-k_f H_r^n\Big|_{r=r_b} = Q^n = \frac{2(H_b - H^n\Big|_{r=r_b})}{\sqrt{(2\alpha/k_c^0)(H_b - H^{n-1}\Big|_{r=r_b}) + \beta_0^{-2}} + \beta_0^{-1}},$$

$$\frac{\partial H^n}{\partial z}\Big|_{z=0,L_z} = 0, \qquad H^n\Big|_{r=L} = 0$$

(τ is the regularization parameter).

It should be noted that the equations for formation fluid transport and salt concentration cannot be written in terms of time-integrated functions. However, the mud filtrate volumes obtained for various reservoir strata using the stationary model allow estimates of the average characteristics of the formation-fluid displacement fronts.

The nonstationary model (1)–(6) and the stationary model (7)–(9) were used to solve three problems of straight borehole drilling with the following data common for all versions. The modeling domain ($L = 40$ m and $L_z = 12$ m), which represented an isolated reservoir, was partitioned by a difference grid cells with 61 nodes along the radius and 31 along the vertical coordinate. The nonpenetration condition was specified on the formation top and bottom, and a constant pressure equal to the initial formation pressure $h_f = 0$ was assumed on the right boundary ($r = L$). The grid step was variable along the radius (the minimum value near the borehole was equal to 0.03 m), and it was constant along the vertical coordinate z. The reservoir 12 m thick consisted of five layers (z_i was the depth of the lower boundary of the ith stratum), which had the following hydrophysical characteristics:

1) $z_1 = 3$ m, $k = 10$ millidarcy, and $m_0 = 0.22$;
2) $z_2 = 6$ m, $k = 20$ millidarcy, and $m_0 = 0.20$;
3) $z_3 = 8$ m, $k = 5$ millidarcy, and $m_0 = 0.19$;
4) $z_4 = 10$ m, $k = 10$ millidarcy, and $m_0 = 0.16$;
5) $z_5 = 12$ m, $k = 30$ millidarcy, and $m_0 = 0.18$.

The compressibility of all strata was identical: $\varepsilon = 10^{-5}$ atm^{-1}. The borehole drilling conditions were specified as follows: before the time $T - 0.05$, the excess borehole pressure head was $h_b = 300$ m (drilling time) in the time interval $[T - 0.05, T]$, $h_b = 0.01$ m (downtime without drilling mud circulation). The production process parameters were a drilling rate of 200 m/day and a borehole diameter of 0.2 m. The parameters of the formation fluids were as follows: water viscosity 1 cP $\approx 10^{-3}$ Pa$\cdot$ sec, salt concentration in the formation water $C_f = 20$ g/liter, and salt concentration in the drilling mud $C_b = 1$ g/liter. The mud cake permeability was set equal to $k_c = 4 \cdot 10^{-3}$ millidarcy, and $\alpha = 0.06$. The formation water saturation had different values for different versions.

Example 1. *Oil-Saturated Reservoir.* In each of the five strata of the reservoir, the initial water saturation was specified as follows: 1) $S_f = 0.3$; 2) $S_f = 0.33$; 3) $S_f = 0.35$; 4) $S_f = 0.42$; 5) $S_f = 0.38$. The exponents in the expressions for the relative permeabilities were $n_1 = n_2 = 3$, and the ratio of the viscosities of the formation water and oil was $\mu_0 = 0.3$. At the time $t = 0$, the borehole bottom was below the formation bottom: $l_b(0) > L_z$.

The numerical calculations were performed for the nonstationary and stationary models. Two values of the borehole drilling times were considered: $T = 0.30$ and 0.55 day. A comparison was performed of the calculation results (mud cake thickness and integrated-pressure distributions in the near-borehole zone at the times T) obtained for both models. The calculated thicknesses of the mud cake formed during drilling are given in Table 1. Figure 1 shows isolines of the integrated pressure head H for $T = 0.55$ day.

TABLE 1

Mud Cake Thickness d [mm]
Calculated for the Monstationary and Stationary Models (Example 1)

z, m	Nonstationary model		Stationary model		Water-saturated reservoir ($T = 0.55$ day)
	$T = 0.3$ day	$T = 0.55$ day	$T = 0.3$ day	$T = 0.55$ day	
0	4.262	7.307	4.251	7.310	8.269
0.8	4.370	7.446	4.254	7.313	8.270
1.6	4.385	7.466	4.265	7.325	8.272
2.4	4.420	7.503	4.298	7.360	8.279
3.2	4.587	7.660	4.505	7.573	8.319
4.0	4.615	7.686	4.534	7.603	8.324
4.8	4.602	7.670	4.524	7.590	8.321
5.6	4.531	7.594	4.454	7.511	8.305
6.4	4.006	7.077	3.775	6.777	8.173
7.2	3.797	6.822	3.606	6.557	8.121
8.0	2.547	5.517	2.029	4.550	7.680
8.8	1.954	4.663	1.670	3.883	7.461
9.6	2.530	5.455	2.036	4.471	7.629
10.4	4.734	7.799	4.670	7.726	8.342
11.2	4.802	7.872	4.742	7.814	8.361
12.0	4.813	7.884	4.751	7.826	8.364

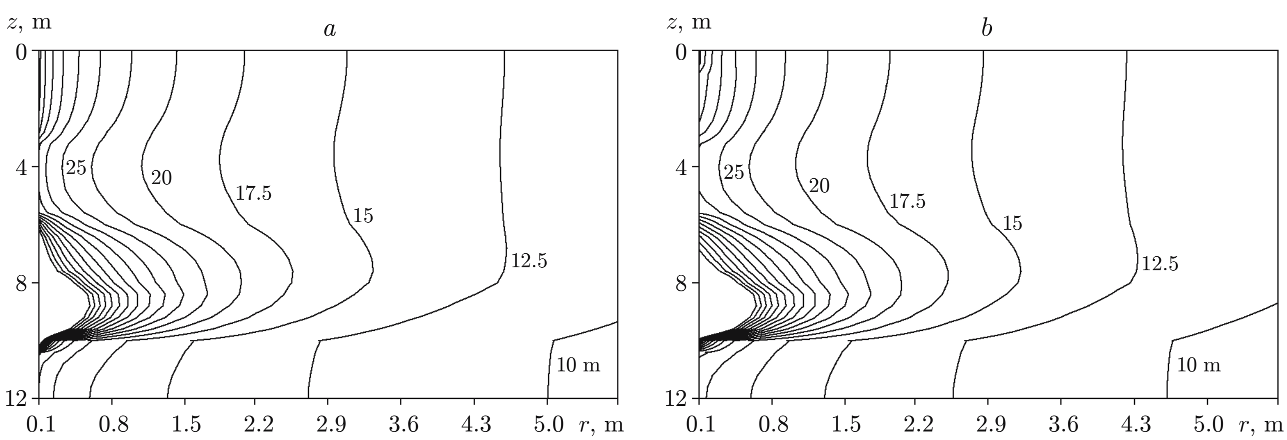

Fig. 1. Distribution of the integrated pressure head H in the near-borehole zone for $T = 0.55$ day (Example 1): (a) stationary model; (b) nonstationary model.

For poorly permeable strata ($k < 10$ millidarcy), the flow coefficients of the stratum and mud cake are close; in this case, therefore, the mud cake formation model is sensitive to permeability variations, which influences the calculated values of the mud cake thickness for stratum 4 at $z = 8$–10 m (see Table 1). For highly permeable strata, the permeability variation has a weak effect on the mud cake thickness.

It should be noted that the presence of strata with different hydrophysical characteristics leads to the occurrence of flow of complex structure in the near-borehole zone. A increase in the mud cake thickness equalizes the volumes of mud filtrate invasion into different strata. However, unloading of the filtration flow in the system of hydrodynamically coupled formations occurs primarily over highly permeable strata, which results in exchange vertical overflows between strata with different permeabilities. At a distance from the borehole equal to several tens of meters, the pressure difference between the strata is almost completely equalized and the formation-fluid flow becomes nearly radial.

The stationary model allows predictions of the volume of mud filtrate invasion into a reservoir and exchange flows between neighboring strata.

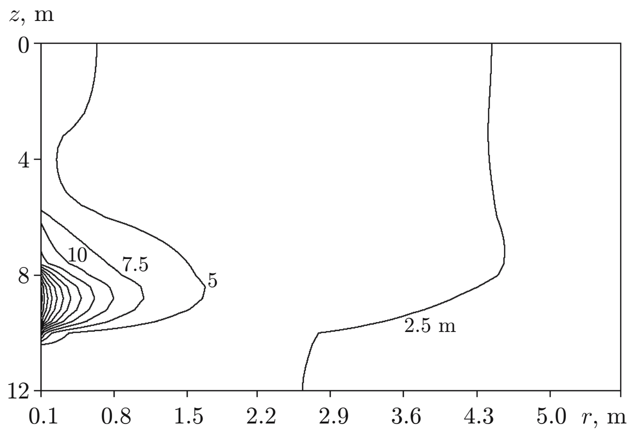

Fig. 2. Distribution of the integrated pressure head H in the near-borehole zone for $T = 0.55$ day (Example 2; stationary model).

Example 2. *Water-Saturated Reservoir.* In the case of a water-saturated reservoir, only the salt concentration distribution changes in the invaded zone. If clogging processes occur only in a narrow zone near the borehole wall, the rock permeability does not change during drilling. In this case, the nonstationary and stationary models should predict identical values for the mud cake thickness and integrated pressure head. Let us consider a water-saturated reservoir in which oil is absent in all strata: $S_f = 1$. Numerical modeling shows that, in the reservoir, the mud cake thickness varies in the range of 7.461–8.364 mm (see Table 1). The difference between the mud cake thickness calculated for the stationary and nonstationary models is less than 0.0005 mm, which is within the numerical error. Figure 2 gives the distribution of the integrated pressure head in the near-borehole zone obtained for the stationary model, which coincides for the distribution obtained for the nonstationary model. Thus, in the case of a water-saturated reservoir, the results calculated for both models coincide.

In Example 2, the pressure head in the near-borehole zone is much lower than the values obtained for the problem with an oil-saturated reservoir (see Example 1). This is due to the fact that, in the presence of oil, the reservoir permeability decreases sharply compared to the permeability of a water-saturated reservoir. In this case, the mud filtrate invasion into the reservoir decreases, resulting in a decrease in the thickness of the mud cake formed during drilling of the oil-saturated reservoir.

Example 3. *Accounting for the Drilling Process in a Reservoir.* We consider a water-saturated reservoir in which oil is absent in all reservoir strata: $S_f = 1$. At the initial time, the borehole bottom is at the reservoir top level: $l_b(0) = 0$. The borehole drilling rate is 200 m/day.

In the stationary model, to approximately take into account the drilling process in the reservoir, we specify the depth-varying integrated pressure head in the borehole:

$$H_b = \int_{z/v_b}^{T} h_b \, dt.$$

It is assumed that the main effect on the region of the reservoir located at a depth z begins at the time $t_1(z) = z/v_b$, i.e., at the moment the bottom hole depth is below this level.

The drilling time was $T = 0.55$ day. Table 2 gives calculated thicknesses of the mud cake formed in a time $t = 0.25$ day and during the entire period of drilling T. Figure 3 shows isolines of the integrated pressure head H calculated for the stationary and nonstationary models. The presence of exchange flows between the strata leads to a complex redistribution of salt concentrations in the invaded zone. Figure 4 gives salt concentration distributions in the near-borehole zone calculated for the nonstationary model.

For $t = 0.25$ day, the reservoir pressure differs from the initial formation pressure, and, hence, for the stationary model, the condition $\Phi = 0$ is not satisfied. However, the error in determining the mud cake thickness is insignificant compared to the calculation using the nonstationary model.

A comparison of the calculations results taking into account reservoir drilling (Example 3) and ignoring drilling (Example 2) shows, that, at high drilling rates, this factor has an insignificant effect. However, at low

TABLE 2

Mud Cake Thicknesses d [mm], Calculated for the Nonstationary and Stationary Models (Example 3)

z, m	Nonstationary model		Stationary model	
	$t = 0.25$ day	$t = 0.55$ day	$t = 0.25$ day	$t = 0.55$ day
0	5.793	8.272	5.790	8.271
0.8	5.748	8.240	5.742	8.238
1.6	5.702	8.209	5.695	8.205
2.4	5.660	8.181	5.652	8.178
3.2	5.650	8.187	5.642	8.183
4.0	5.606	8.158	5.597	8.153
4.8	5.554	8.121	5.543	8.115
5.6	5.490	8.072	5.476	8.065
6.4	5.311	7.906	5.294	7.897
7.2	5.212	7.821	5.192	7.810
8.0	4.737	7.350	4.710	7.334
8.8	4.477	7.095	4.451	7.081
9.6	4.579	7.224	4.558	7.212
10.4	5.209	7.893	5.197	7.888
11.2	5.175	7.878	5.161	7.871
12.0	5.126	7.847	5.107	7.837

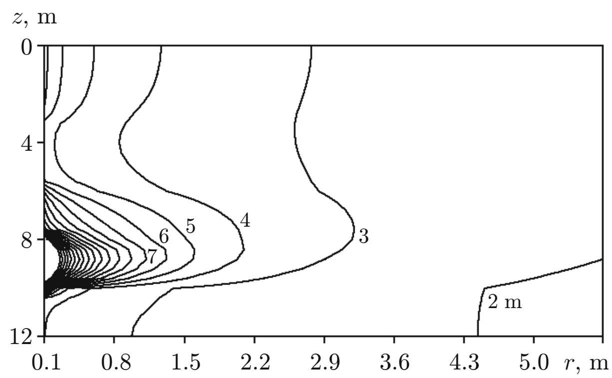

Fig. 3. Distribution of the integrated pressure H in the near-borehole zone for $T = 0.55$ day (Example 3).

drilling rates (for example, in core drilling) the hydrodynamic processes in the bottom hole zone have a significant effect on the formation of the invaded zone.

Determining Invasion Depths for a Water-Saturated Reservoir. For water-saturated reservoirs, the invasion depth R_n in the middle strata of a reservoir can be estimated assuming frontal displacement of formation fluids by drilling mud filtrate. The specific filtrate volume V is determined from the mud cake thickness d_c in the middle part of the reservoir:

$$R_n = \sqrt{2V/m + r_b^2}, \qquad V = r_b Q = r_b \int_0^T q\,dt = \frac{r_b d_c}{\alpha}.$$

At the same time, for radial displacement, the position of the middle concentration isoline $C_c = (C_b + C_f)/2$ is close to the position of the salt concentration front for the complete displacement of formation water. Using the calculation results for the nonstationary problem (see Fig. 4), it is possible to determine the distance from the middle concentration isoline to the center of the borehole R_p.

In Table 3, the calculated values (R_p) and theoretical values (R_n) of the invasion depth for Example 2 are in fairly good agreement. The calculated specific volume of filtrate V for different strata varies in a narrow range. However, in the presence of oil, the specific filtrate volume for poorly permeable strata decreases sharply compared to moderately and highly permeable strata.

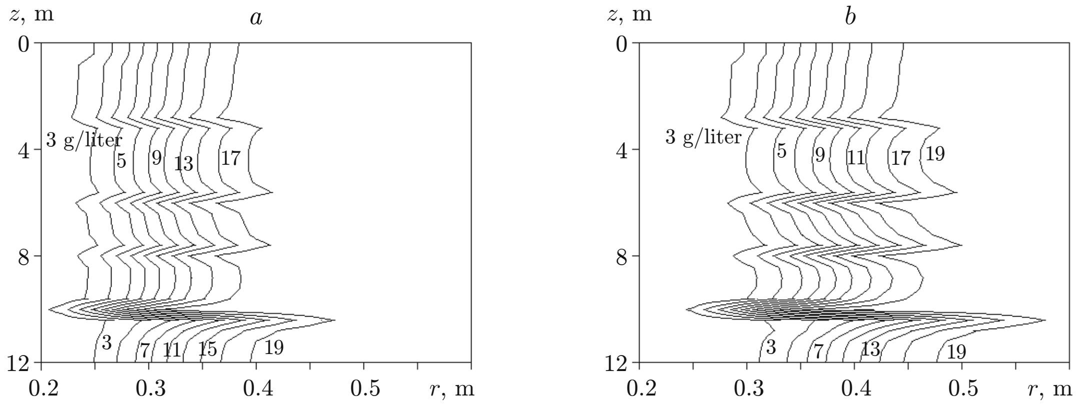

Fig. 4. Salt concentration distribution C in the near-borehole zone (Example 3; nonstationary model): (a) $t = 0.25$ day; (b) $t = 0.55$ day.

TABLE 3

Estimation of Invasion Depth (Example 2)

Strata number	$t = 0.25$ day				$t = T = 0.55$ day			
	d_c, mm	V, m^2	R_n, m	R_p, m	d_c, mm	V, m^2	R_n, m	R_p, m
1	5.70	0.00950	0.310	0.304	8.21	0.01367	0.366	0.365
2	5.57	0.00928	0.321	0.315	8.14	0.01157	0.383	0.380
3	5.22	0.00870	0.319	0.315	7.83	0.01305	0.384	0.384
4	4.46	0.00742	0.320	0.310	7.09	0.01182	0.397	0.385
5	5.17	0.00862	0.325	0.324	7.88	0.01313	0.395	0.398

The invasion depth over the filtrate volume can be estimated only for the middle parts of fairly thick reservoir strata. Because of vertical overflows on the boundaries between strata, the condition of radial displacement of formation fluids is violated. Near the boundary between strata with different permeabilities, there is a characteristic nonmonotonic redistribution of formation fluids according to the direction of exchange flows. Recording the presence of such zones of fluid exchange by borehole logging, one can determine the permeability ratio of neighboring strata.

The above method for the approximate estimation of the invasion depth is also appropriate for oil-saturated reservoirs. In this case, it is necessary to use self-similar solutions of the problem of radial displacement of formation fluids with the filtrate volume calculated for the stationary model.

The approach proposed here to estimate the borehole effect on formations during drilling is easily extended to the three-dimensional case, which makes it suitable for modeling slant-hole and extended-reach drilling.

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